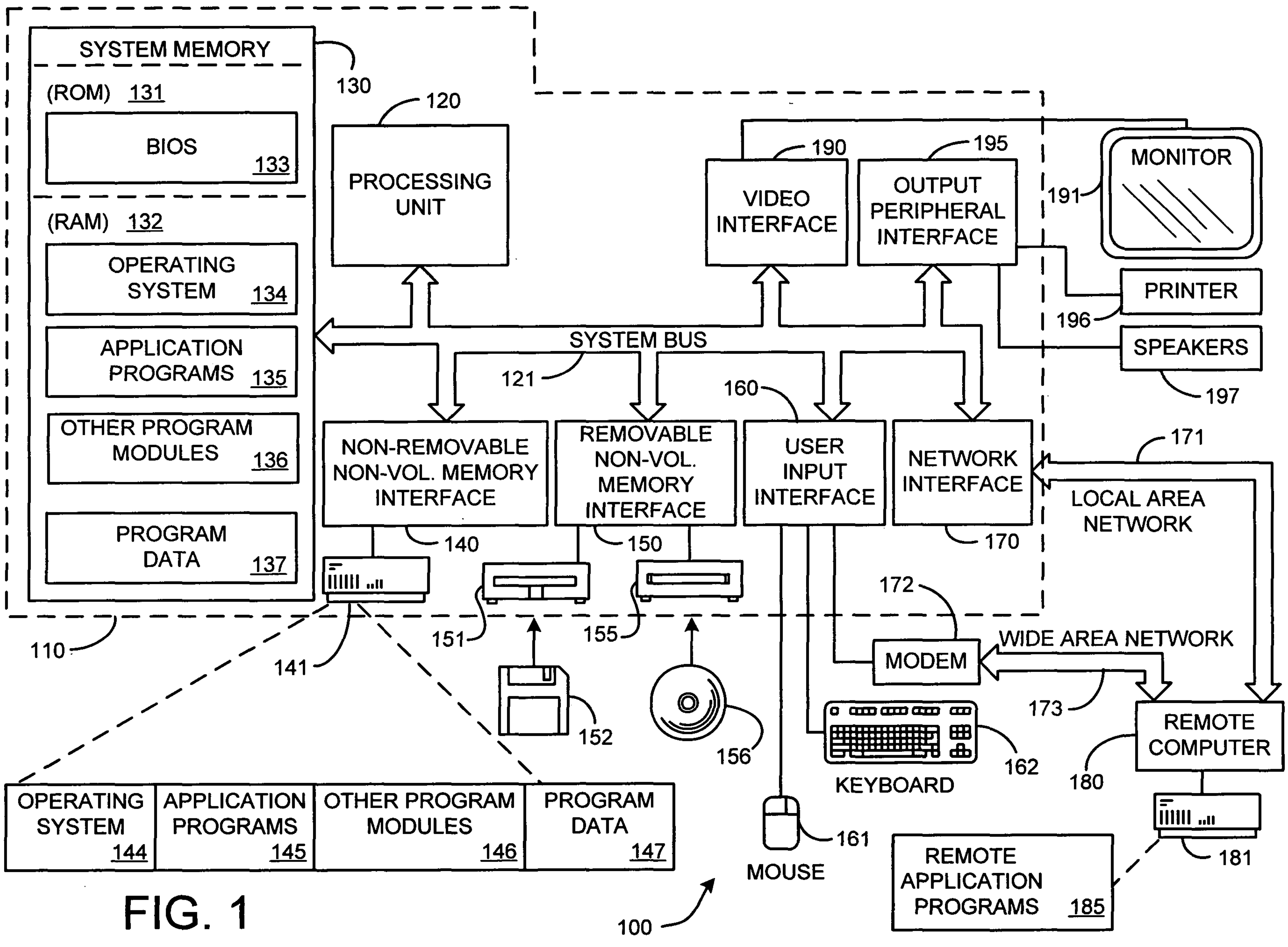
SYSTEM MEMORY
130
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FIG. 1

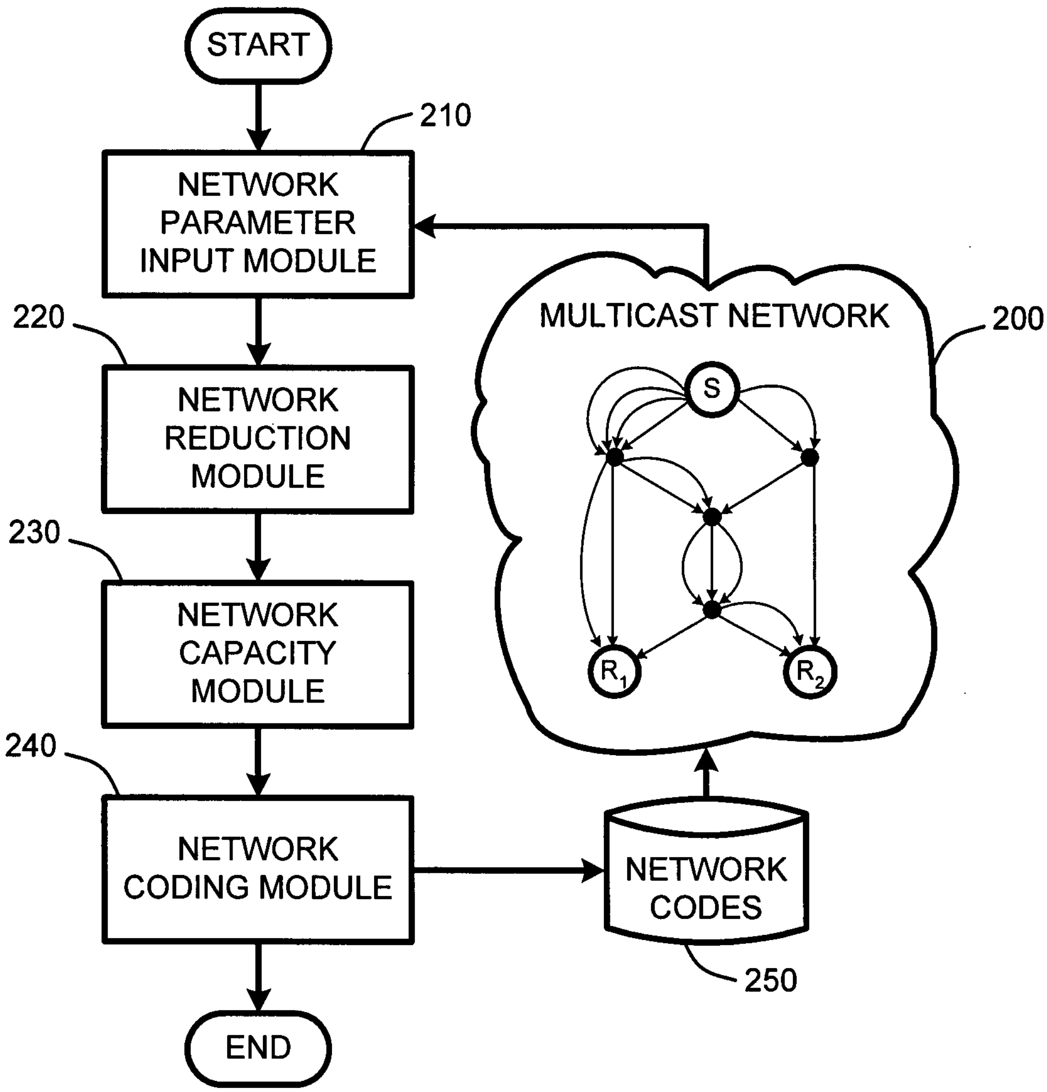

START
210
NETWORK PARAMETER INPUT MODULE
220
NETWORK REDUCTION MODULE
230
NETWORK CAPACITY MODULE
240
NETWORK CODING MODULE
END
MULTICAST NETWORK
200
S
R1
R2
NETWORK CODES
250

FIG. 2

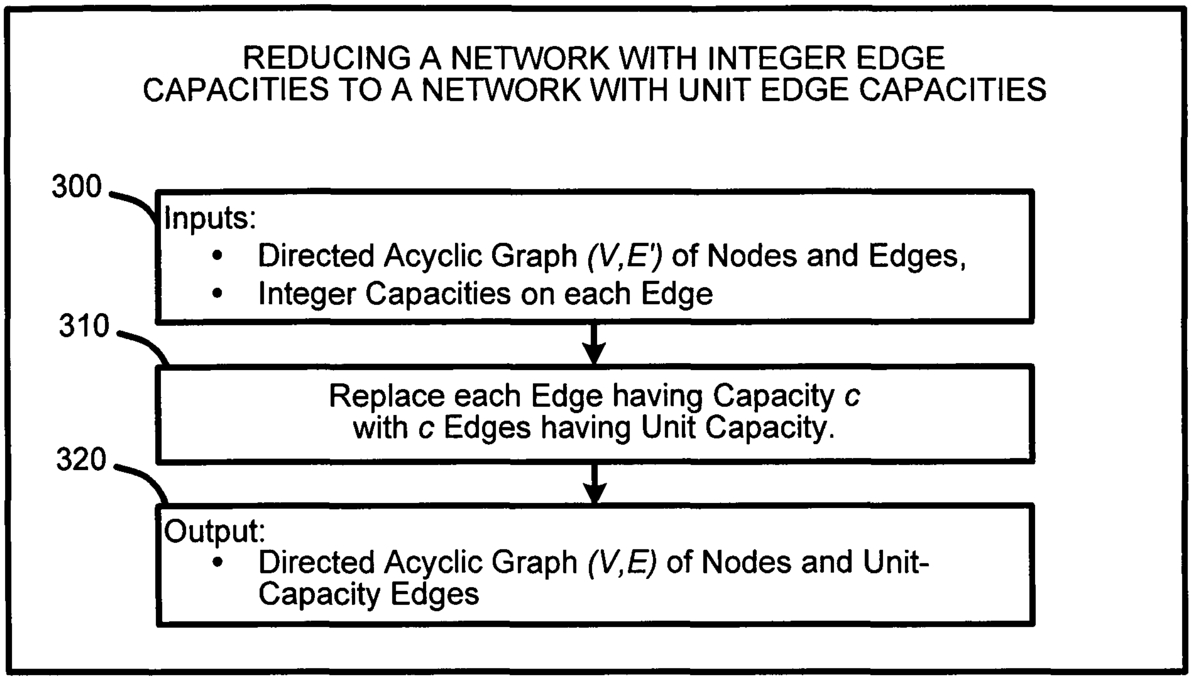

REDUCING A NETWORK WITH INTEGER EDGE CAPACITIES TO A NETWORK WITH UNIT EDGE CAPACITIES
300
Inputs:
• Directed Acyclic Graph (V,E') of Nodes and Edges,
• Integer Capacities on each Edge
310
Replace each Edge having Capacity c with c Edges having Unit Capacity.
320
Output:
• Directed Acyclic Graph (V,E) of Nodes and Unit-Capacity Edges

FIG. 3

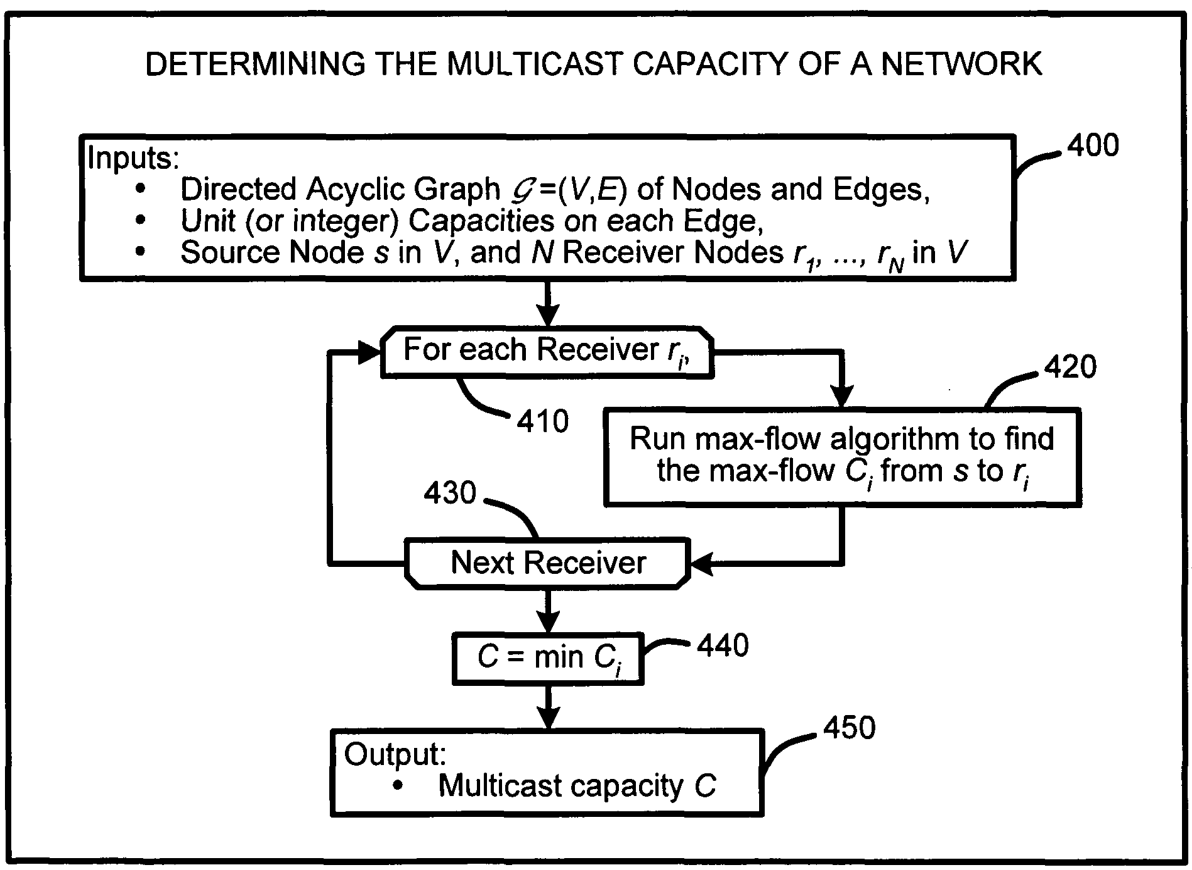
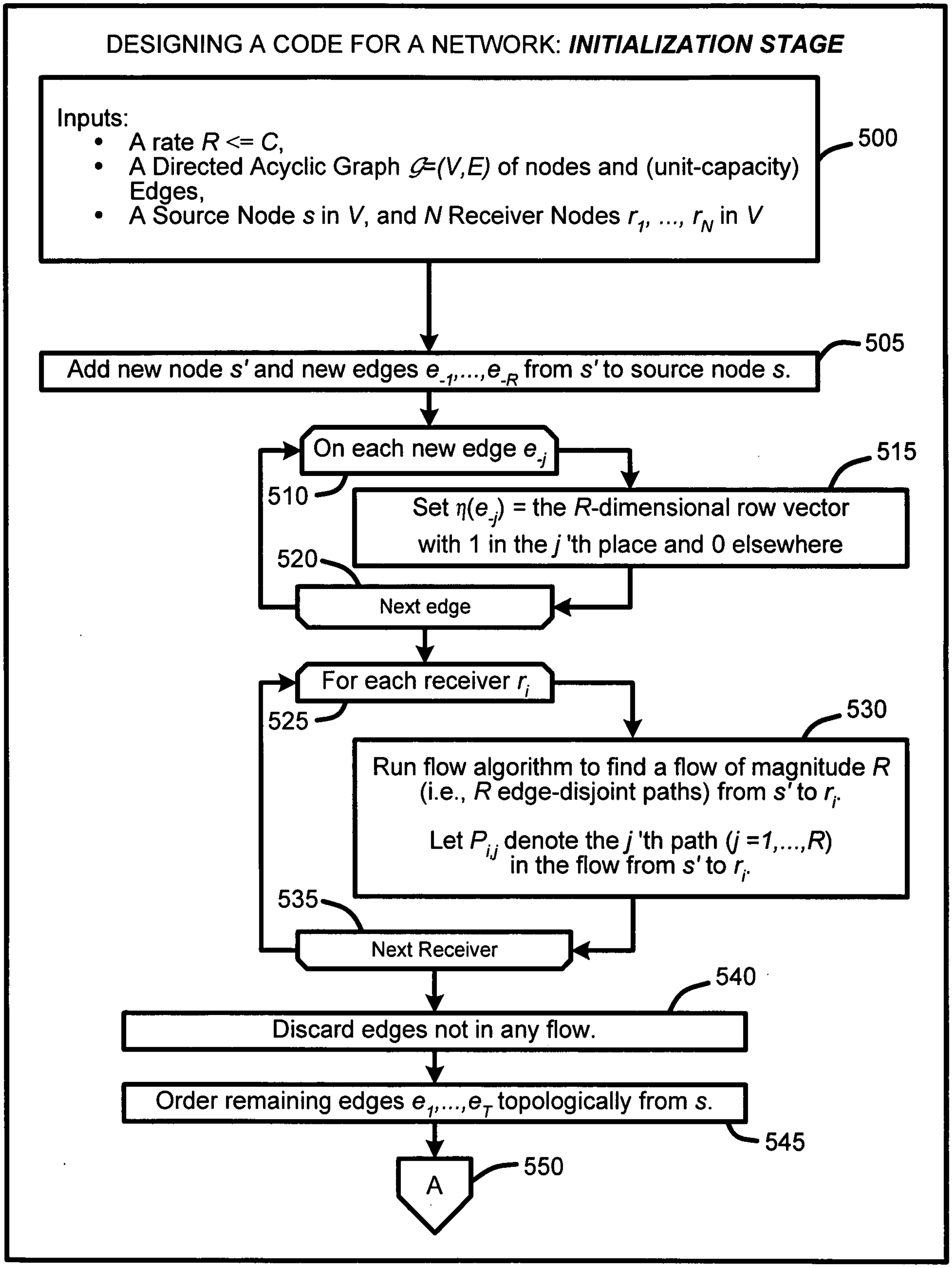
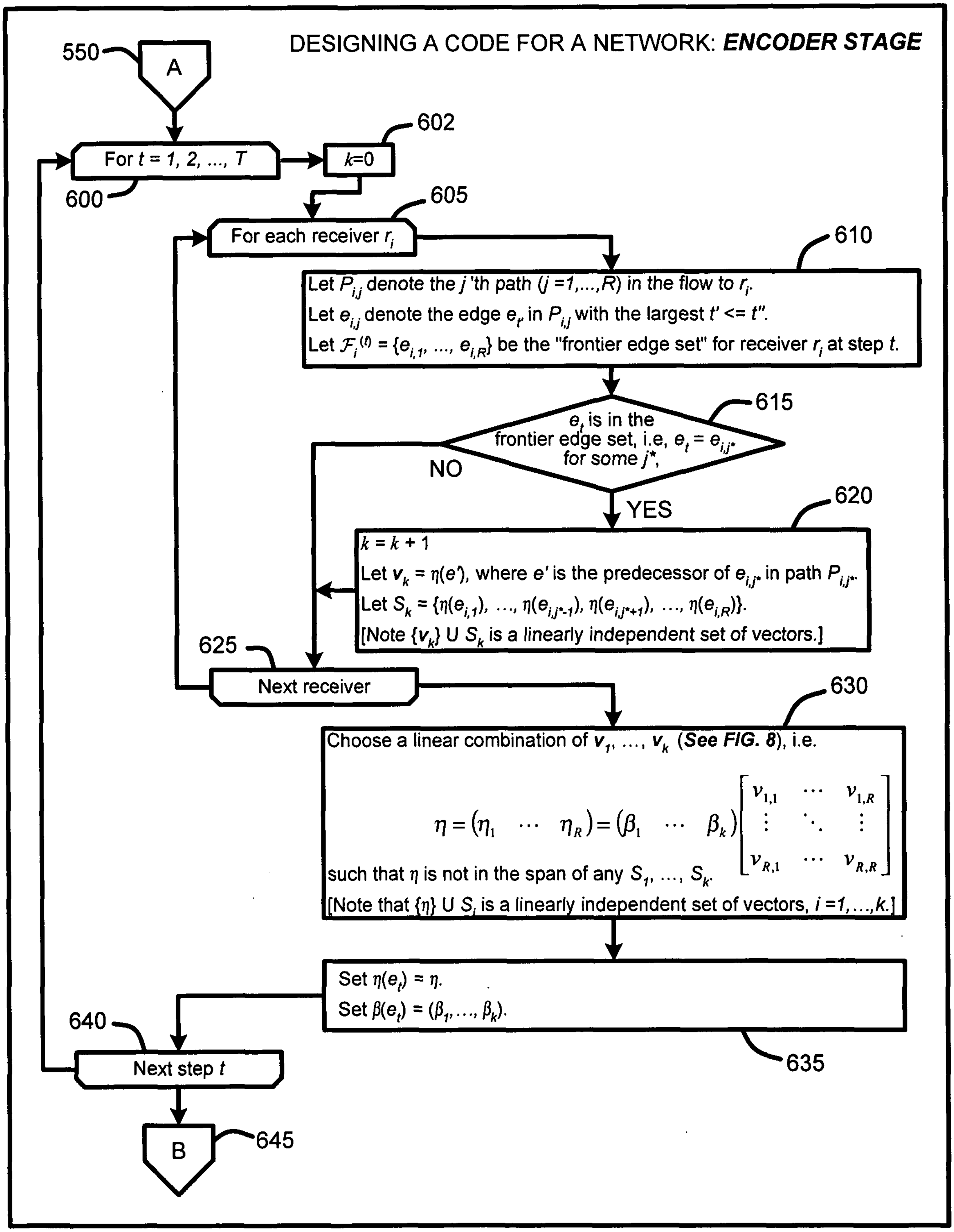

DETERMINING THE MULTICAST CAPACITY OF A NETWORK
Inputs:
• Directed Acyclic Graph $\mathcal{G}$=(V,E) of Nodes and Edges,
• Unit (or integer) Capacities on each Edge,
• Source Node s in V, and N Receiver Nodes r_1, ..., r_N in V
400
For each Receiver r_i,
410
420
Run max-flow algorithm to find the max-flow C_i from s to r_i
430
Next Receiver
C = min C_i
440
450
Output:
• Multicast capacity C

FIG. 4



DESIGNING A CODE FOR A NETWORK: INITIALIZATION STAGE
Inputs:
• A rate $R <= C$,
• A Directed Acyclic Graph $\mathcal{G}=(V,E)$ of nodes and (unit-capacity) Edges,
• A Source Node s in V, and N Receiver Nodes $r_1, ..., r_N$ in V
500
Add new node s' and new edges $e_{-1},...,e_{-R}$ from s' to source node s.
505
On each new edge e_{-j}
510
Set $\eta(e_{-j})$ = the R-dimensional row vector with 1 in the j 'th place and 0 elsewhere
515
520
Next edge
For each receiver r_i
525
Run flow algorithm to find a flow of magnitude R (i.e., R edge-disjoint paths) from s' to r_i.
Let $P_{i,j}$ denote the j 'th path ($j=1,...,R$) in the flow from s' to r_i.
530
535
Next Receiver
540
Discard edges not in any flow.
Order remaining edges $e_1,...,e_T$ topologically from s.
545
A
550

FIG. 5



DESIGNING A CODE FOR A NETWORK: ***ENCODER STAGE***
550
A
For t = 1, 2, ..., T
600
k=0
602
For each receiver r_i
605
610
Let $P_{i,j}$ denote the j'th path (j =1,...,R) in the flow to r_i.
Let $e_{i,j}$ denote the edge $e_{t'}$ in $P_{i,j}$ with the largest $t' <= t$".
Let $\mathcal{F}_i^{(t)} = \{e_{i,1}, ..., e_{i,R}\}$ be the "frontier edge set" for receiver r_i at step t.
615
e_t is in the frontier edge set, i.e, $e_t = e_{i,j^*}$ for some j^*,
NO
YES
620
$k = k + 1$
Let $\mathbf{v}_k = \eta(e')$, where e' is the predecessor of e_{i,j^*} in path P_{i,j^*}.
Let $S_k = \{\eta(e_{i,1}), ..., \eta(e_{i,j^*-1}), \eta(e_{i,j^*+1}), ..., \eta(e_{i,R})\}$.
[Note $\{\mathbf{v}_k\}$ U S_k is a linearly independent set of vectors.]
625
Next receiver
630
Choose a linear combination of $\mathbf{v}_1$, ..., $\mathbf{v}_k$ (***See FIG. 8***), i.e.
$$\eta = (\eta_1 \ \cdots \ \eta_R) = (\beta_1 \ \cdots \ \beta_k)\begin{bmatrix} v_{1,1} & \cdots & v_{1,R} \\ \vdots & \ddots & \vdots \\ v_{R,1} & \cdots & v_{R,R} \end{bmatrix}$$
such that η is not in the span of any S_1, ..., S_k.
[Note that $\{\eta\}$ U S_i is a linearly independent set of vectors, i =1,...,k.]
Set $\eta(e_t) = \eta$.
Set $\beta(e_t) = (\beta_1, ..., \beta_k)$.
635
640
Next step t
B
645

FIG. 6

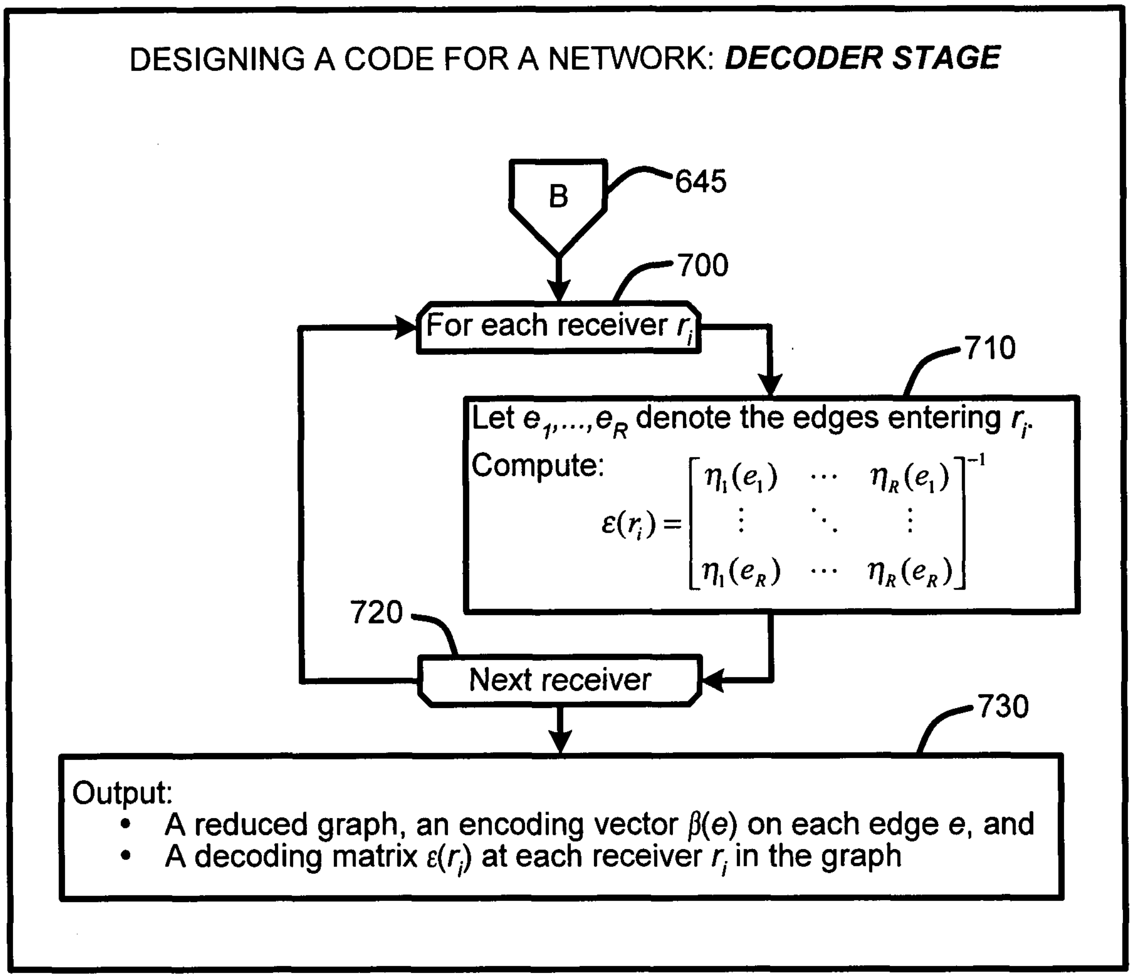

DESIGNING A CODE FOR A NETWORK: ***DECODER STAGE***
B
645
700
For each receiver r_i
710
Let $e_1,\ldots,e_R$ denote the edges entering r_i.
Compute:
$$\varepsilon(r_i) = \begin{bmatrix} \eta_1(e_1) & \cdots & \eta_R(e_1) \\ \vdots & \ddots & \vdots \\ \eta_1(e_R) & \cdots & \eta_R(e_R) \end{bmatrix}^{-1}$$
720
Next receiver
730
Output:
• A reduced graph, an encoding vector $\beta(e)$ on each edge e, and
• A decoding matrix $\varepsilon(r_i)$ at each receiver r_i in the graph

FIG. 7

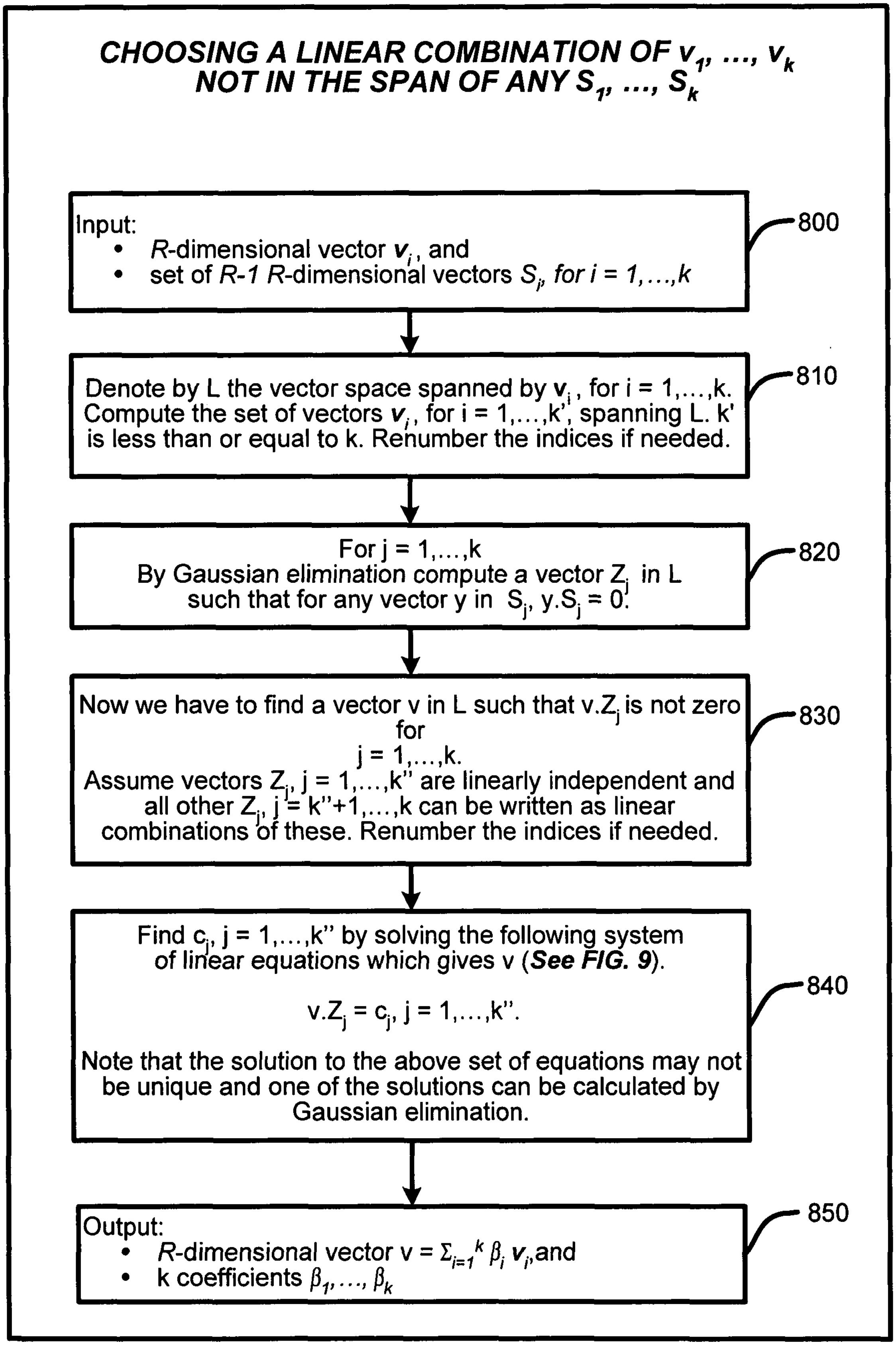

CHOOSING A LINEAR COMBINATION OF $v_1, \ldots, v_k$ NOT IN THE SPAN OF ANY $S_1, \ldots, S_k$
Input:
• R-dimensional vector v_i, and
• set of R-1 R-dimensional vectors S_i, for $i = 1,\ldots,k$
800
Denote by L the vector space spanned by v_i, for $i = 1,\ldots,k$. Compute the set of vectors v_i, for $i = 1,\ldots,k'$, spanning L. k' is less than or equal to k. Renumber the indices if needed.
810
For $j = 1,\ldots,k$
By Gaussian elimination compute a vector Z_j in L such that for any vector y in S_j, $y.S_j = 0$.
820
Now we have to find a vector v in L such that $v.Z_j$ is not zero for $j = 1,\ldots,k$.
Assume vectors Z_j, $j = 1,\ldots,k''$ are linearly independent and all other Z_j, $j = k''+1,\ldots,k$ can be written as linear combinations of these. Renumber the indices if needed.
830
Find c_j, $j = 1,\ldots,k''$ by solving the following system of linear equations which gives v (See FIG. 9).
$v.Z_j = c_j$, $j = 1,\ldots,k''$.
Note that the solution to the above set of equations may not be unique and one of the solutions can be calculated by Gaussian elimination.
840
Output:
• R-dimensional vector $v = \Sigma_{i=1}^{k} \beta_i v_i$, and
• k coefficients $\beta_1, \ldots, \beta_k$
850

FIG. 8

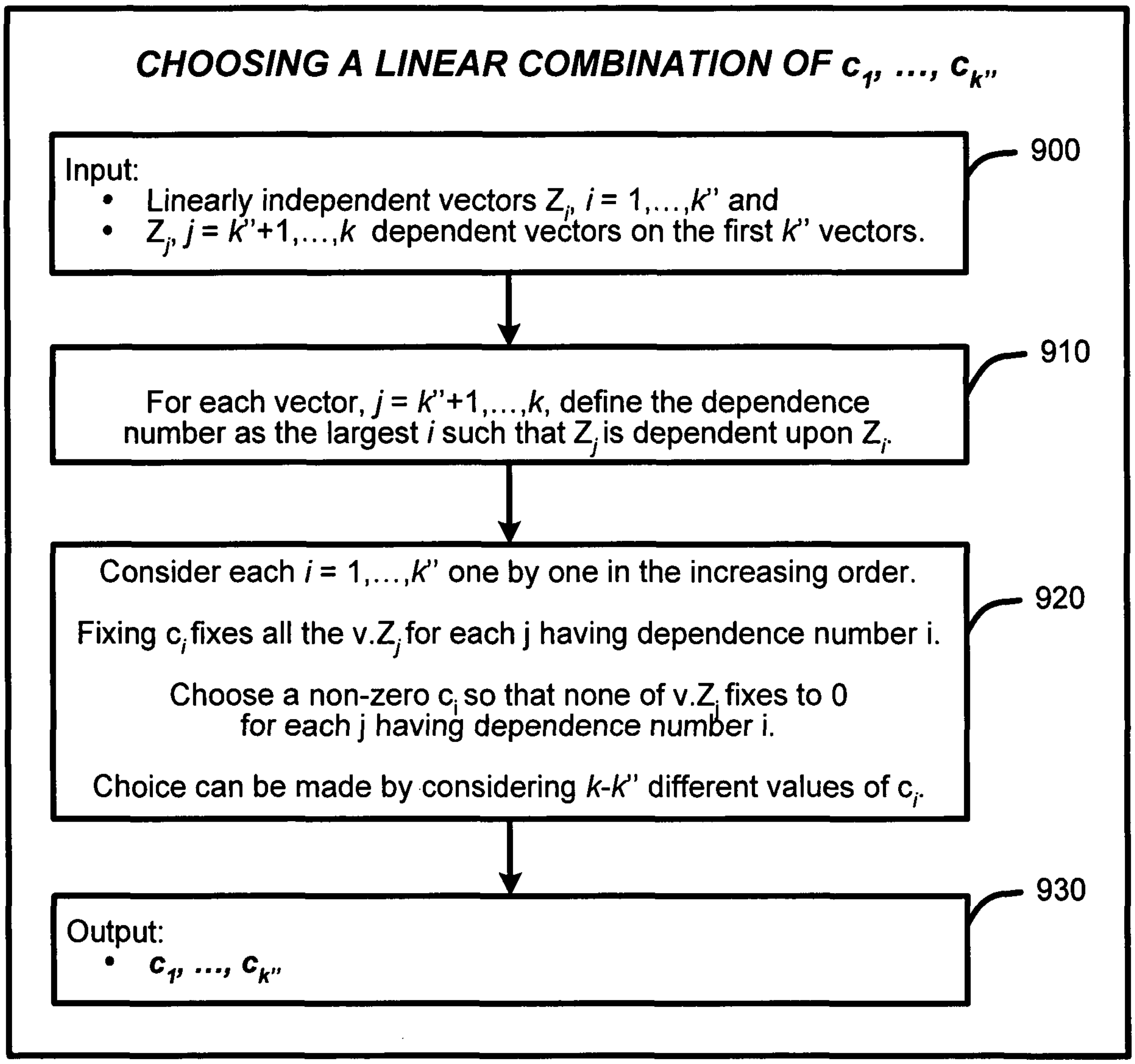

CHOOSING A LINEAR COMBINATION OF $c_1, \ldots, c_{k''}$
Input:
• Linearly independent vectors Z_i, $i = 1,\ldots,k'$ and
• Z_j, $j = k'+1,\ldots,k$ dependent vectors on the first k' vectors.
900
For each vector, $j = k'+1,\ldots,k$, define the dependence number as the largest i such that Z_j is dependent upon Z_i.
910
Consider each $i = 1,\ldots,k'$ one by one in the increasing order.
Fixing c_i fixes all the $v.Z_j$ for each j having dependence number i.
Choose a non-zero c_i so that none of $v.Z_j$ fixes to 0 for each j having dependence number i.
Choice can be made by considering k-k' different values of c_i.
920
Output:
• $c_1, \ldots, c_{k''}$
930

FIG. 9

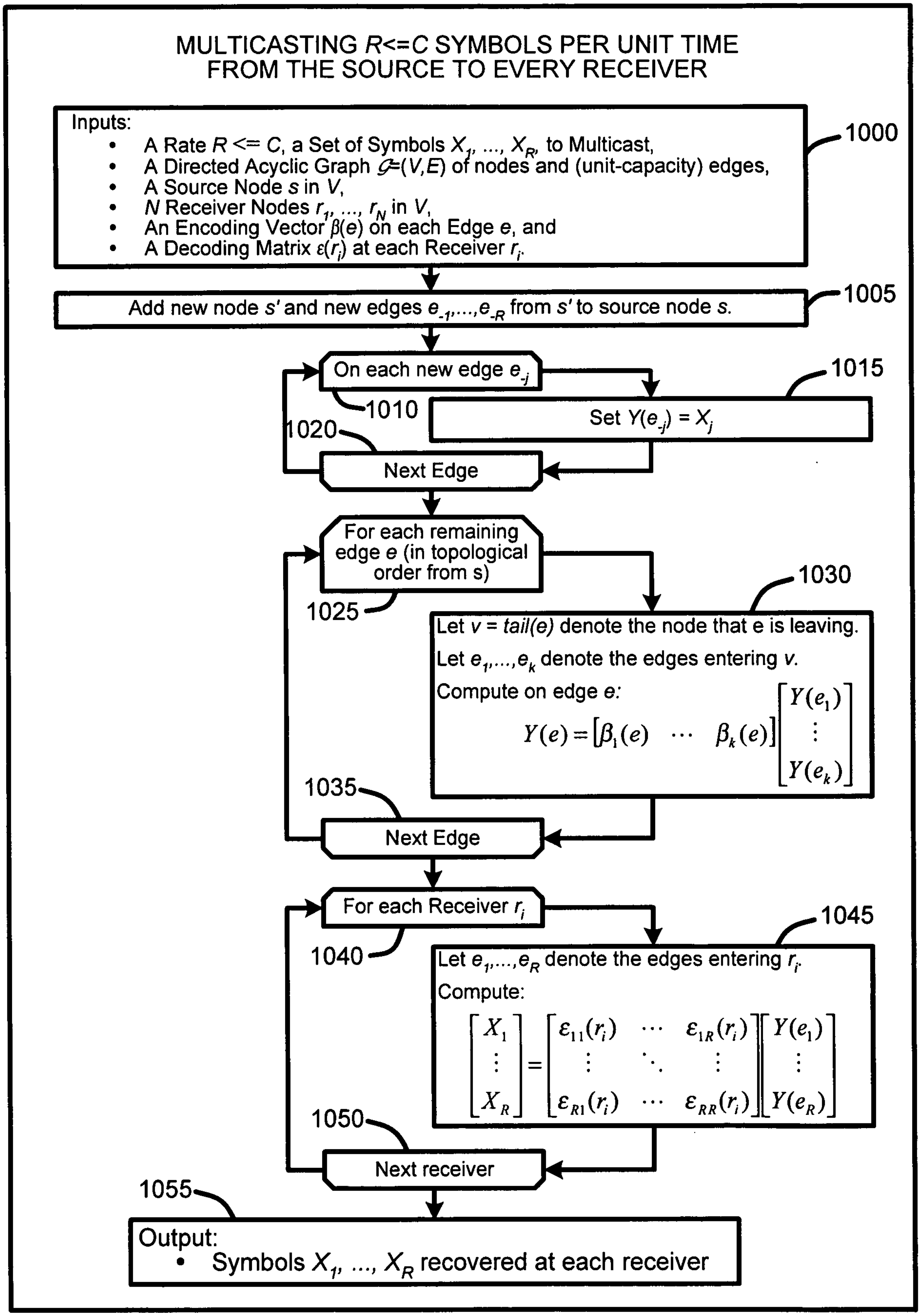

MULTICASTING R<=C SYMBOLS PER UNIT TIME FROM THE SOURCE TO EVERY RECEIVER
Inputs:
• A Rate R <= C, a Set of Symbols X_1, ..., X_R, to Multicast,
• A Directed Acyclic Graph $\mathcal{G}$=(V,E) of nodes and (unit-capacity) edges,
• A Source Node s in V,
• N Receiver Nodes r_1, ..., r_N in V,
• An Encoding Vector $\beta(e)$ on each Edge e, and
• A Decoding Matrix $\varepsilon(r_i)$ at each Receiver r_i.
1000
Add new node s' and new edges $e_{-1},\ldots,e_{-R}$ from s' to source node s.
1005
On each new edge e_{-j}
1010
Set $Y(e_{-j}) = X_j$
1015
1020
Next Edge
For each remaining edge e (in topological order from s)
1025
1030
Let v = $tail(e)$ denote the node that e is leaving.
Let $e_1,\ldots,e_k$ denote the edges entering v.
Compute on edge e:
$$Y(e)=\begin{bmatrix}\beta_1(e) & \cdots & \beta_k(e)\end{bmatrix}\begin{bmatrix}Y(e_1)\\ \vdots \\ Y(e_k)\end{bmatrix}$$
1035
Next Edge
For each Receiver r_i
1040
1045
Let $e_1,\ldots,e_R$ denote the edges entering r_i.
Compute:
$$\begin{bmatrix}X_1\\ \vdots \\ X_R\end{bmatrix}=\begin{bmatrix}\varepsilon_{11}(r_i) & \cdots & \varepsilon_{1R}(r_i)\\ \vdots & \ddots & \vdots \\ \varepsilon_{R1}(r_i) & \cdots & \varepsilon_{RR}(r_i)\end{bmatrix}\begin{bmatrix}Y(e_1)\\ \vdots \\ Y(e_R)\end{bmatrix}$$
1050
Next receiver
1055
Output:
• Symbols X_1, ..., X_R recovered at each receiver

FIG. 10